Antimagic Forests

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Known Result

Antimagic Labeling

Antimagic Labeling
Let $G = (V, E)$ be a graph. Let $f\colon E \to \{1, 2, 3, \ldots, |E|\}$ be an *edge-labeling*. Let $\phi_f(v) = \Sigma_{v \quad E(v)} f(e)$ be the *vertex-sum*. An edge-labeling f is an antimagic labeling if f is bijective and each vertex has a unique vertex-sum.

Antimagic Conjecture
Every connected graph other than K_2 is antimagic.

Known Theorem [1]

Lemma
Let s, l be non-negative integers and let $k = 2s + 6l$. Then there is a partition of $A = \{1, 2, \ldots, k\}$ into subsets $A_1, A_2, \ldots, A_{s+2l}$ such that each subset is: a 2-set whose elements sum to $k + 1$, a 3-set whose elements sum to $k + 1$, or a 3-set whose elements sum to $2(k + 1)$.

Example $s = 1, l = 2, k = 2(1) + 6(1) = 8$

1	2	3	4
8	7	6	5

1	2	4
3	7	6
5		8

Corollary
Let $k = r_1 + r_2 + \cdots + r_t$ be a partition of the positive integer k, where $r_i \geq 2$ for $i = 1, 2, \ldots, t$. We can partition $\{1, 2, \ldots, k\}$ into pairwise disjoint subsets $A_1, A_2, \ldots, A_t$ such that for every $1 \leq i \leq t$, $|A_i| = r_i$ and $\Sigma_{a \quad A_i} a \equiv 0 \pmod{k\ }$ where $k \ = k + 1$ of k is even, and $k \ = k$ if k is odd.

Theorem
If $T = K_2$ is a tree with at most one vertex of degree 2, then T is antimagic.

Antimagic Labeling Strategy of a Forest

Let F be a forest with component trees $T_1, T_2, \ldots, T_s$ and at most one degree-2 vertex. Depending on s and the existence of a degree-2 vertex, our labeling strategy is modified. The most general labeling strategy will be described below.

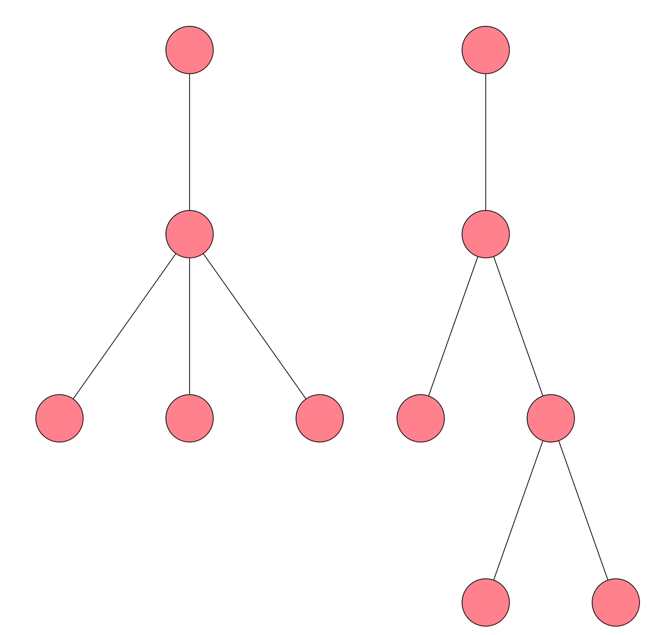
Root each component T_i at a leaf and orient from parent to child. Denote the root of T_i as w_i. Let T be the tree obtained by identifying $w_1, w_2, \ldots, w_s$ as a single vertex w. Then we use the corollary to label the edges of T. The corollary makes the outgoing edge labels of a vertex be some multiple of $|E(T)|$ or $|E(T)| + 1$. After assigning the labels to T, we split T,with the labels, back into the components of F.

Forest

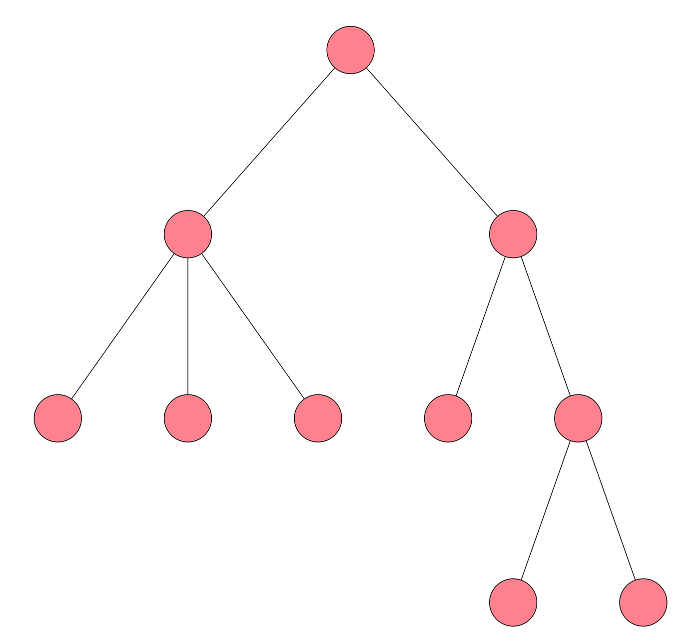
(Disjoint Components)

Tree

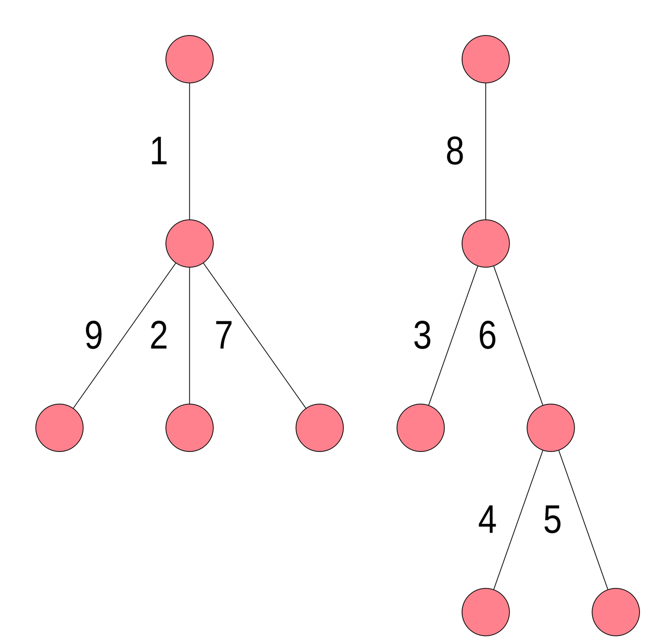
(Components Joined)



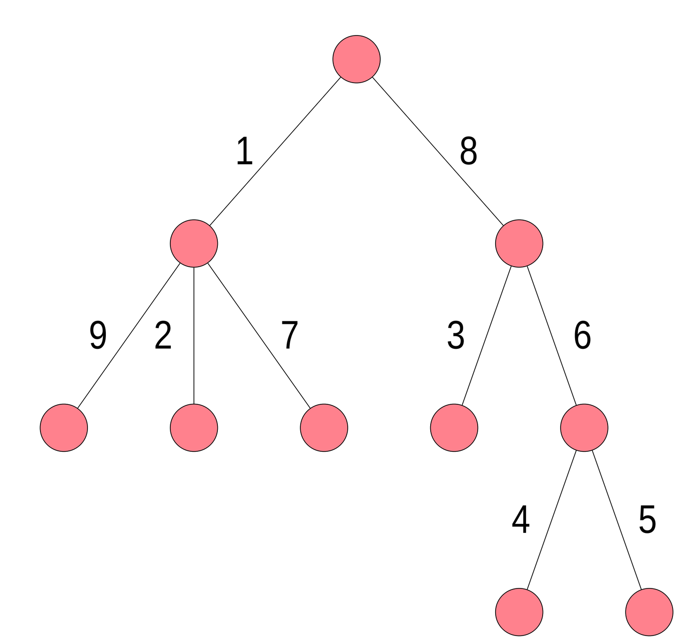
Step 1: Root components at leaves



Step 2: Combine roots to create T



Step 4: Split T back into F



1	2	3	4
8	6	5	7

Step 3: Label T using corollary

New Result

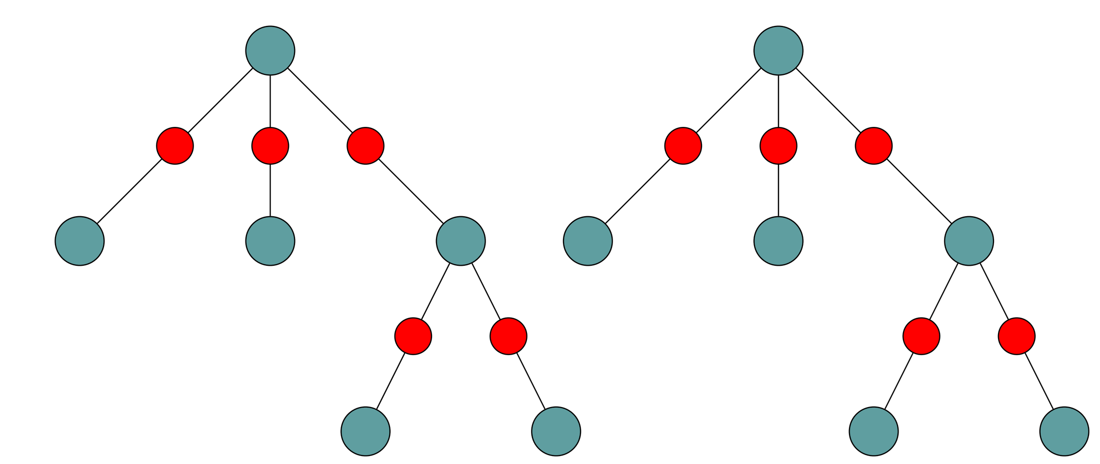
New Theorem

(Extension to forests)

Let F be a forest with components $T_1, T_2, \ldots, T_s$. If F has at most one degree-2 vertex then F is antimagic

Future Work

Question: *Can we find an antimagic labeling for forests with more than one degree 2 vertex?*



Subdivided Forest

Reference

[1] Yu-Chang Liang, Tsai-Lien Wong, Xuding Zhu. *Anti-magic labeling of trees*. Discrete Mathematics 331 (2014) 9-14.

Acknowledgments

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