Understanding of Mathematical Induction

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Abstract

Many undergraduates have difficulty with the principle of mathematical induction (PMI). This project seeks to understand the common struggles associated with this topic and how it relates to students' overall performance in their course. Additionally, it explores how educators can transform their standard approach to teaching mathematical induction in a way that builds on students' prior knowledge to create a strong foundation for this type of proof writing.

Background

Theorem 1 (Principle of Mathematical Induction)
Let P(n) be a statement about natural numbers. If
(1) We find numbers for which P(n) is true and
(2) Assume n = k and P(k) is true to show
P (k+1) is also true using the induction hypothesis,
Then P(n) is true for all n in the natural numbers.

Methodology

This project gathered data from fifty students, a majority of whom were freshmen. We analyze their response to a midterm question on mathematical induction requiring knowledge of series:

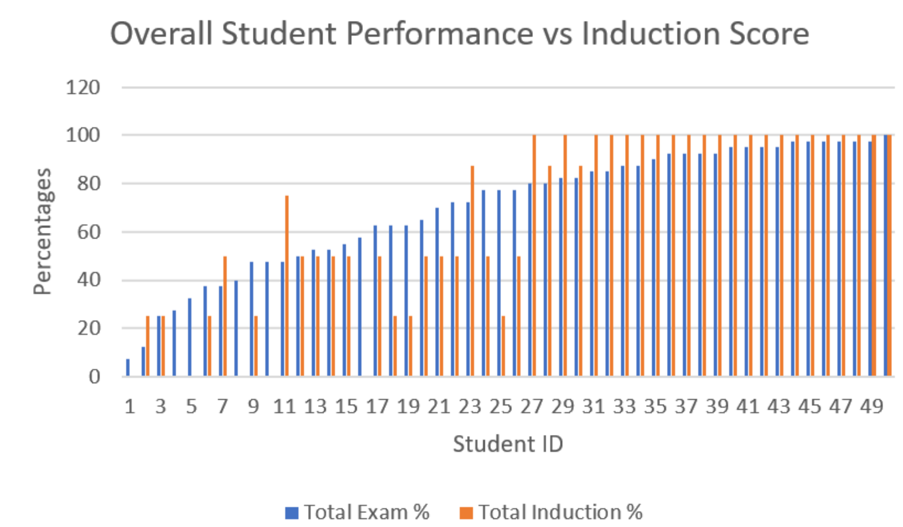
Prove using mathematical induction that the sum of the first numbers divisible by 4 is given by the formula:

$$4 + 8 + 12 + \ldots + 4n = 2n\,(n+1)$$

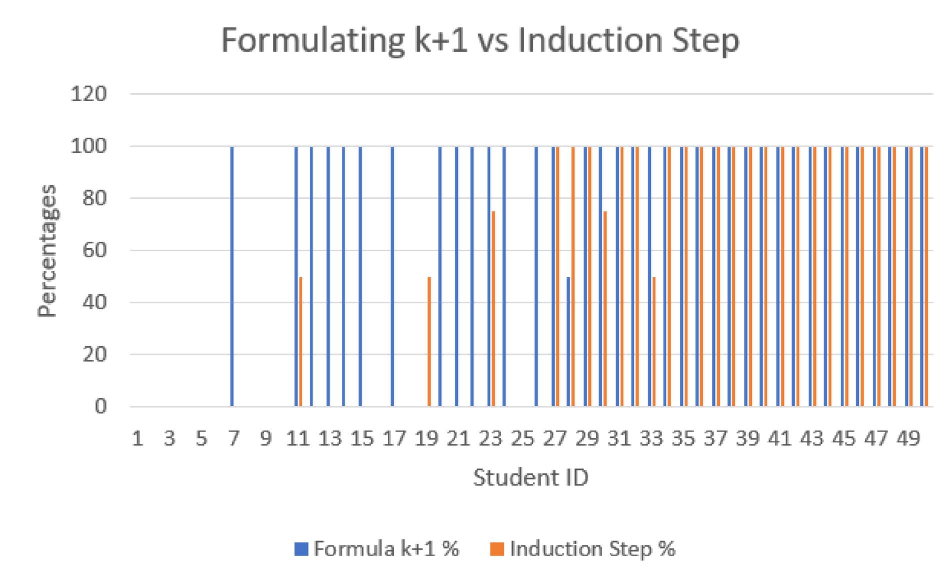
Our data explored several correlations between student understanding of checking for small numbers, formulating k + 1 correctly, identifying the induction hypothesis, performing the induction step and arriving at correct conclusions.

Results

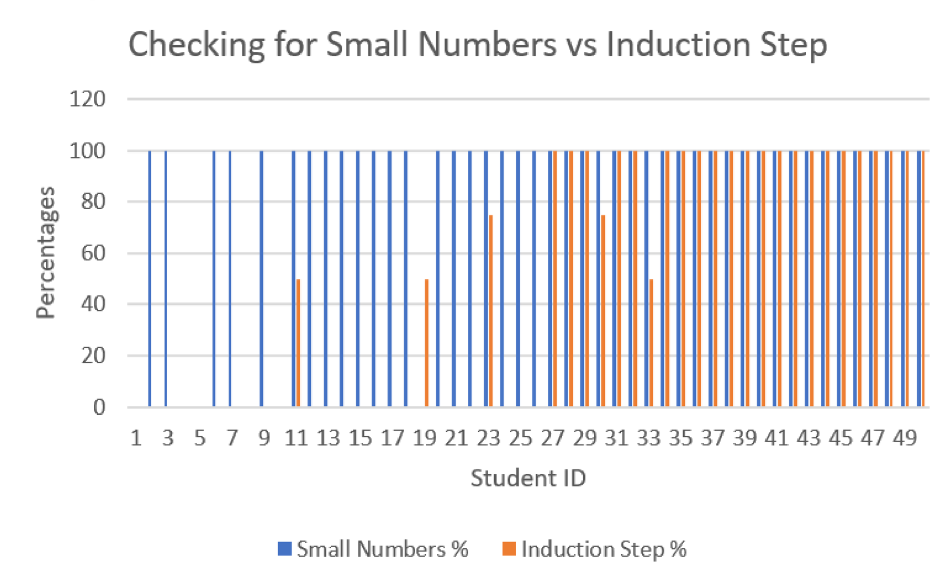
We found strong correlations between overall student performance and their score on mathematical induction. Students who did well overall on the midterm were more likely to complete the induction problem correctly.



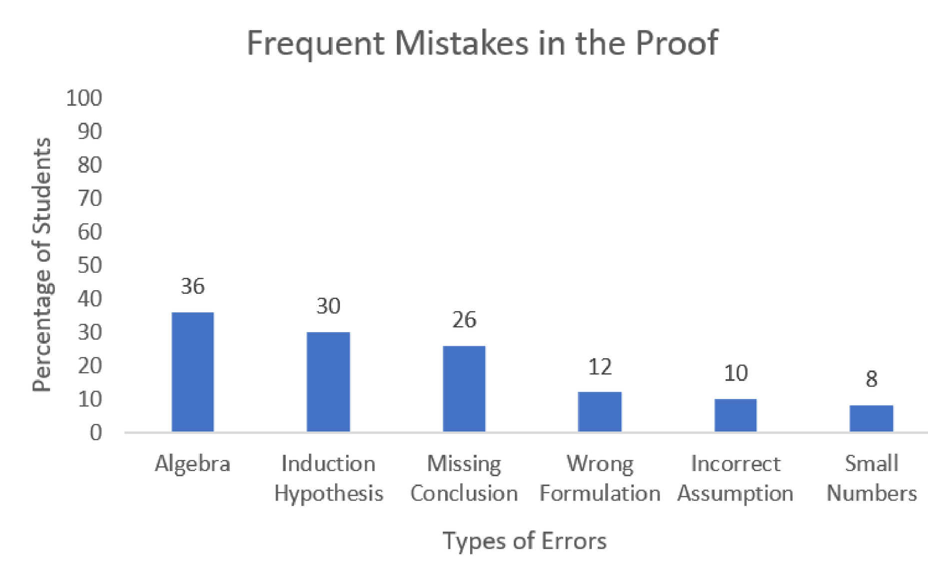
Student understanding of checking for small numbers had little correlation with doing the induction step correctly.



If students could formulate k + 1 correctly, they had a slightly higher chance of doing the induction step correctly.



Finally, we found that a majority of students made algebraic mistakes and did not correctly utilize the induction hypothesis.



Future Work

Before introducing mathematical induction, it may be helpful to review topics such as series, sigma notation and solving algebraic expressions using deductive reasoning. Additionally, the use of analogies such as the domino effect can create a strong foundation for student understanding and provide a smooth transition into induction proofs. We can explore the effects of this technique in future work.

Concepts to Review

1. Expand and solve: $\sum_{n=1}^{4}(n-1) =$
2. Expand: $\sum_{n=1}^{n} 3n =$
3. Write in sigma notation.
   $2 + 6 + 18 + 54 =$
4. Write in sigma notation.
   $8 + \frac{8}{3} + \frac{8}{9} + \cdots\cdots + \frac{8}{3^n} =$
5. Solve $5 + \sqrt{x+14} = x + 7$ using deductive reasoning.
   (Note: Deductive reasoning means using definitions, properties, and logic to form a logical argument)
6. Imagine you place several dominoes close to one another in a straight line. What happens when you push the first domino? What events follow? Do you see any patterns? How can you represent this pattern?

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